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Strongly Lacunary δ-quasi-Cauchy sequences in 2-normed spaces

Sibel Ersan

Maltepe University, Marmara Eğitim Köyü, TR 34857, Maltepe, Istanbul-Turkey

sibelersan@maltepe.edu.tr

Abstract. A sequence (x_k) of points in a subset E of a 2-normed space X is called strongly lacunary δ-quasi-Cauchy, or N_θ-δ-quasi-Cauchy if (Δx_k) is N_θ-convergent to 0, that is $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} \|\Delta^2 x_k, z\| = 0$ for every fixed $z \in X$. A function defined on a subset E of X is called strongly lacunary δ-ward continuous if it preserves N_θ-δ-quasi-Cauchy sequences, i.e. $(f(x_k))$ is an N_θ-δ-quasi-Cauchy sequence whenever (x_k) is. In this study we obtain some theorems related to strongly lacunary δ-quasi-Cauchy sequences.

Keywords: Strongly lacunary ward continuity, quasi-Cauchy sequences, continuity, 2-normed space
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INTRODUCTION

A lacunary sequence $\theta = (k_r)$ is an increasing sequence of positive integers such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$, the ratio k_r/k_{r-1} will be abbreviated by q_r, $q_1 = 0$ for convention, and we assume that $\lim inf_r\ q_r > 1$. In [1], the concept of a strongly lacunary convergent sequence of real numbers, or an N_θ convergent sequence, was defined by Freedman, Sember, and Raphael. A sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary convergent to a real number L or N_θ-convergent to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} |\alpha_k - L| = 0$, and it is denoted by $N_\theta - \lim \alpha_k = L$. Using the idea of Freedman, Sember, and Raphael; Fridy and Orhan introduced the concept of lacunary statistical convergence of a sequence of real numbers in [2, 3]. A sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically convergent, or S_θ-convergent, to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |\alpha_k - L| \geq \varepsilon\}| = 0$ for every positive real number ε, it is denoted by $S_\theta - lim_{k\to\infty} \alpha_k = L$.

In recent years many kinds of continuities were introduced and investigated ([4, 5, 6]). A sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary quasi-Cauchy if $N_\theta - lim \Delta\alpha_k = 0$, where $\Delta\alpha_k = \alpha_{k+1} - \alpha_k$ for each positive integer k ([7, 8, 9, 10, 11, 12]). The set of strongly lacunary quasi-Cauchy sequences in $\mathbb{R}$ will be denoted by ΔN_θ. A function defined on a subset A of $\mathbb{R}$ is called strongly lacunary ward continuous or N_θ-ward continuous if it preserves N_θ-quasi-Cauchy sequences of points in A, i.e. $(f(\alpha_k))$ is N_θ-quasi-Cauchy whenever (α_k) is an N_θ-quasi-Cauchy sequence of points in A. Recently, the concept of the ward continuity in 2-normed spaces was investigated in [13, 14, 15].

The purpose of this paper is to introduce the concept of strongly lacunary delta ward continuity in 2-normed spaces and prove some related theorems.

Strongly Lacunary δ-ward continuity

A sequence (x_k) of points in a subset E of a 2-normed space X is called strongly lacunary quasi-Cauchy, or N_θ-quasi-Cauchy if (Δx_k) is N_θ-convergent to 0, that is

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} \|\Delta x_k, z\| = 0$$

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for every fixed $z \in X$ and it is denoted by $N_\theta - \lim_{k\to\infty} \|\Delta x_k, z\| = 0$. A function defined on a subset E of X is called strongly lacunary ward continuous if it preserves N_θ-quasi-Cauchy sequences, i.e. $(f(x_k))$ is an N_θ-quasi-Cauchy sequence whenever (x_k) is [15].

Now we introduce strongly lacunary delta quasi-Cauchy sequence in 2-normed space X in the following.

Definition 1 *A sequence (x_k) of points in a subset E of X is called strongly lacunary delta quasi Cauchy, or N_θ-δ quasi Cauchy if the sequence (Δx_k) is an N_θ quasi Cauchy sequence, that is*

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} \|\Delta^2 x_k, z\| = 0$$

for each positive integer k and for any fixed $z \in X$.

Throughout this paper, N_θ, $\Delta N_\theta(x)$ and $\Delta^2 N_\theta(x)$ will denote the set of strongly lacunary convergent sequences, strongly lacunary quasi-Cauchy sequence and strongly lacunary delta quasi-Cauchy sequence in X respectively.

Moreover the subsequence of the N_θ-δ quasi-Cauchy sequence need not to be a N_θ-δ quasi-Cauchy. Now we introduce the concept of N_θ-δ ward compactness of a subset of X.

Definition 2 *If any sequence of points in a subset E has an N_θ-δ quasi-Cauchy subsequence, then E is called strongly lacunary delta ward compact, or N_θ-δ-ward compact.*

Any finite subset of X is N_θ-δ-ward compact. A union of two N_θ-δ-ward compact subsets of X is N_θ-δ-ward compact and also the intersection of any N_θ-δ-ward compact subsets of X is N_θ-δ-ward compact.

In the following we introduce a definition of N_θ-δ ward continuity in X.

Definition 3 *A real valued function f defined on a subset E of X is called N_θ-δ ward continuous if it preserves N_θ-δ quasi-Cauchy sequences of points in E, in other words; $(\Delta f(x_k))$ is a N_θ-δ quasi-Cauchy sequence whenever (Δx_k) is a N_θ-δ quasi-Cauchy sequence of points in E.*

The set of N_θ-δ ward continuous functions on E will be denoted by $\Delta^2 N_\theta(E)$.

Proposition 1 *The set of N_θ-δ ward continuous functions is a vector space.*

If a function f is N_θ-δ ward continuous on a subset E of the 2-normed space X, then it is N_θ ward continuous on E. But the converse of the statement is not true. Here is the proof of the statement.

Theorem 1 *If a function f is N_θ-δ ward continuous on a subset E of the 2-normed space X, then it is N_θ ward continuous on E.*

Moreover if a function is N_θ-δ ward continuous on a subset E of X, then it is easily verified that is also ordinary continuous on E. Proof can be easily verified by using the previous theorem and the Theorem 3.9 in [13].

Theorem 2 *The function f is uniformly continuous on a subset E of X. If (x_n) is any quasi-Cauchy sequence of points in E, then the sequence $(f(x_n))$ is a N_θ-δ quasi-Cauchy.*

Theorem 3 *If (f_n) is a sequence of N_θ-δ ward continuous functions on a subset E of X and (f_n) is uniformly convergent to a function f, then f is also N_θ-δ ward continuous on E.*

Theorem 4 *The set of N_θ-δ ward continuous functions on a subset E of X is a closed subset of the set of continuous functions on E.*

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