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Letter

Born–Infeld-$f(R)$ black holes

Salih Kibaroğlu

Faculty of Engineering and Natural Sciences, Maltepe University, Istanbul, 34857, Türkiye

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ABSTRACT

We explore black hole solutions in the context of Born-Infeld-$f(R)$ gravity, a modified gravitational framework that extends both Born-Infeld and $f(R)$ theories. By adopting a static, spherically symmetric spacetime ansatz, we derive an exact black hole solution and investigate its geometrical structure. We proceed to analyze the thermodynamic properties of the solution, including the Hawking temperature, entropy, and specific heat, with particular emphasis on their dependence on the model parameters. Our results reveal novel thermodynamic behavior that deviates significantly from the standard predictions of general relativity. A comparative study with the Schwarzschild-AdS black holes is also presented, showing how Born-Infeld-$f(R)$ corrections alter black hole thermodynamics.

1. Introduction

The study of black hole solutions within modified theories of gravity has gained substantial interest in recent years, largely due to their potential to address foundational problems in cosmology, high-energy physics, and quantum gravity. Among the prominent alternatives to general relativity (GR), Born-Infeld (BI) gravity [1–6] stands out as a compelling candidate. Originally motivated by nonlinear electrodynamics as a means to regularize field singularities [1], the gravitational extension of the BI action introduces a square-root determinant structure that naturally incorporates higher-order curvature corrections. This feature enables the theoretical avoidance of curvature singularities, such as those appearing in black hole interiors [2].

However, the inclusion of higher-order curvature terms often leads to the emergence of ghost modes—unphysical degrees of freedom that signal instability at the quantum level. To circumvent this problem, the Palatini formulation of BI gravity—drawing inspiration from Eddington's purely affine gravity framework [3] has been developed. In this approach, the metric and connection are treated as independent variables, effectively avoiding higher-order derivatives in the field equations and thereby eliminating the ghost problem [4–6]. This formulation is widely referred to as Eddington-inspired Born-Infeld (EiBI) gravity. EiBI gravity has proven to be a fertile ground for addressing gravitational phenomena beyond the scope of GR. It has been successfully applied to early-universe cosmology [7–14], astrophysical modeling [7], black hole physics [15–18], and wormhole configurations [19–21], showcasing its versatility and robustness as a modified gravity framework.

Complementing this approach, $f(R)$ gravity constitutes a well-established generalization of GR, where the Ricci scalar in the Einstein-Hilbert action is replaced with a more general function $f(R)$ [22–24]. This modification has been extensively studied for its implications in late-time cosmic acceleration, dark energy modelling, and high-energy gravitational phenomena. The unification of EiBI gravity with $f(R)$ extensions, resulting in BI-$f(R)$ gravity, yields a rich theoretical structure that accommodates both nonlinear geometric deformations and deviations from Einsteinian dynamics [25]. This model allows for a more comprehensive description of gravitational phenomena which holds promise for resolving some of the outstanding problems in modern cosmology and gravitational theory [25–32] (for a detailed review, see [33]). This presents new avenues for exploring the physics of black holes and cosmological evolution in more general settings.

In this work, we aim to explore static, spherically symmetric black hole solutions within the BI-$f(R)$ framework, focusing on configurations that asymptotically approach Schwarzschild–(Anti)de Sitter (Sch-AdS) geometries. These solutions are not only important for understanding the behavior of black holes in curved cosmological backgrounds but are also of interest in the context of holography and the AdS/CFT correspondence. The presence of a cosmological constant or effective curvature scale in such solutions provides a testing ground for probing gravitational modifications beyond Einstein gravity.

The paper is organized as follows: In Section 2, we provide a brief review of the BI-$f(R)$ gravity theory, outlining its fundamental principles and relevant field equations. In Section 3, we derive a static, spherically symmetric black hole solution from the modified field equations and subsequently analyze its singularity properties through the behavior of the associated curvature invariants. Section 4 is devoted to the thermodynamic analysis of the obtained black hole solution, where we compute and discuss key thermodynamic quantities such as the Hawking

E-mail address: salihkibaroglu@maltepe.edu.tr

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temperature, entropy, and specific heat. Finally, in Section 5, we summarize our findings and discuss possible extensions and implications of the results within the context of modified gravity theories and black hole physics. Throughout this work, we adopt natural units by setting the gravitational constant $8\pi G$, the speed of light c, the reduced Planck constant $\hbar$, and the Boltzmann constant k_B equal to unity.

2. Born-Infeld-$f(R)$ gravity

Let us briefly review the standard BI-$f(R)$ theory [25] (see also [26,27]). In this theory, the original EiBI gravity theory is combined with an additional $F(R)$ that depends on the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$. To avoid any ghost instabilities, the theory is formulated within the Palatini formalism (for more detail see [34,35]), in which the metric $g_{\mu\nu}$ and the connection $\Gamma^{\alpha}_{\beta\gamma}$ are treated as independent variables. The action for this theory is given by

$$S = \frac{1}{\epsilon}\int \mathrm{d}^4x\left[\sqrt{-|g_{\mu\nu}+\epsilon R_{\mu\nu}|}-\lambda\sqrt{-g}\right] + \frac{\alpha}{2}\int \mathrm{d}^4x\left[\sqrt{-g}F(R)\right]+S_m, \tag{1}$$

where the first term represents the standard BI gravitational Lagrangian and the second term is an additional function of the Ricci scalar. S_m is the action associated with the matter fields. The parameter ϵ is a constant with inverse dimensions to that of a cosmological constant, and λ is a dimensionless constant. Note that g is the determinant of the metric.

In the limit as $\epsilon \to 0$, the BI Lagrangian reduces to the standard GR term, and the action simplifies to an $F(R)$ theory. Alternatively, if we take the limit $\alpha \to 0$, we retrieve the BI theory. When we take both limits, GR is naturally recovered. Therefore, the EiBI parameter ϵ bridges these different gravity theories. Note that we assume vanishing torsion and a symmetric Ricci tensor in this formulation.

The field equations can be derived from Eq. (1) through independent variation concerning the metric and connection. The variation of this action with respect to the metric tensor leads to modified metric field equations for the standard BI gravitational model,

$$\frac{\sqrt{-q}}{\sqrt{-g}}q^{\mu\nu}-\left[\lambda-\frac{\alpha\epsilon}{2}F(R)\right]g^{\mu\nu}-\alpha\epsilon F_R R^{\mu\nu}=-\epsilon T^{\mu\nu}, \tag{2}$$

where F_R is the derivative of $F(R)$ with respect to the Ricci scalar, and $T^{\mu\nu}$ is the standard energy-momentum tensor. In the above equations, we have used the notation,

$$q_{\mu\nu} = g_{\mu\nu}+\epsilon R_{\mu\nu}, \tag{3}$$

and we denoted the inverse of $q_{\mu\nu}$ by $q^{\mu\nu}$ and q represents the determinant of $q_{\mu\nu}$. Similarly, the connection $\Gamma^{\alpha}_{\beta\gamma}$ variation has the form;

$$\nabla_\lambda\left(\sqrt{-q}q^{\mu\nu}+\alpha\sqrt{-g}F_R g^{\mu\nu}\right)=0, \tag{4}$$

where the covariant derivative is taken with respect to the independent connection, which is defined for a scalar field ϕ as

$$\nabla_\mu\phi = \partial_\mu\phi - \Gamma^{\alpha}_{\mu\alpha}\phi. \tag{5}$$

Furthermore, it is well established in the literature on Palatini $F(R)$ theories that the independent connection can be expressed in terms of an auxiliary metric $h_{\mu\nu}$, which is conformal to $g_{\mu\nu}$ (for more detail, see [34]). Assuming that $q_{\mu\nu}$ is conformally proportional to the metric tensor as

$$q_{\mu\nu} = p(R)g_{\mu\nu}, \tag{6}$$

where $p(R)$ is a function of the Ricci scalar, and insert this expression into (4), which yields

$$\nabla_\lambda\left[\left(p(R)+\alpha F_R\right)\sqrt{-g}g^{\mu\nu}\right]=0. \tag{7}$$

We can now define an auxiliary metric tensor

$$u_{\mu\nu} = \left(p(R)+\alpha F_R\right)g_{\mu\nu}, \tag{8}$$

such that the above equation reduces to $\nabla_\beta\left[\sqrt{-u}u^{\mu\nu}\right]=0$, here $u^{\mu\nu}$ represents the inverse representation of $u_{\mu\nu}$. In Einstein's theory the connection equation takes exactly this form, $\nabla_\beta\left[\sqrt{-g}g^{\mu\nu}\right]=0$, which establishes the compatibility of the connection with the metric, thus leading to the Levi-Civita connection as a solution (see [36] for details) in the torsionless case. Therefore, in our case, we have

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}\left(u^{-1}\right)^{\rho\sigma}\left(u_{\sigma\nu,\mu}+u_{\mu\sigma,\nu}-u_{\mu\nu,\sigma}\right), \tag{9}$$

and this provides a complete and exact solution of the connection equation.

3. Black hole solution

Black hole solutions play a pivotal role in testing the consistency and physical implications of gravitational theories, especially in modified gravity models where nonlinear and higher-curvature corrections are introduced. In the context of BI-$f(R)$ gravity, exploring static and spherically symmetric black hole configurations allows us to understand how deviations from GR affect space-time near compact objects. In this section, we construct a black hole solution by starting from a suitably chosen ansatz and analyzing the field equations. We begin by assuming a static, spherically symmetric space-time of the form:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2+\frac{1}{f(r)}\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right), \tag{10}$$

where $f(r)$ is the metric function that encapsulates the gravitational structure of the black hole and reveals the influence of both the BI nonlinearity and $F(R)$ gravity corrections.

Using the conformal relation between the auxiliary metric $q_{\mu\nu}$ and the physical metric $g_{\mu\nu}$ defined in Eqs. (3) and (6) , it follows that the Ricci tensor is necessarily proportional to the metric tensor, leading to the condition:

$$R_{\mu\nu}(\Gamma) = \Omega(r)g_{\mu\nu}, \tag{11}$$

where we define $\Omega(r) = \frac{1}{\epsilon}[p(R)-1]$ for simplicity.

On the other hand, by introducing a redefinition of the metric function as

$$u = p(R)+\alpha F_R, \tag{12}$$

the auxiliary metric equation Eq. (8) can be considerably simplified to the form $u_{\mu\nu} = u(r)g_{\mu\nu}$. Within this framework, the auxiliary metric becomes conformally related to the physical metric, and can be explicitly expressed as

$$\mathrm{d}s_u^2 = -u(r)f(r)\mathrm{d}t^2+\frac{u(r)}{f(r)}\mathrm{d}r^2+u(r)r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{13}$$

By utilizing the Ricci tensor components derived from the physical metric Eq. (11), the gravitational field equations yield the following system of coupled nonlinear differential equations:

$$-\frac{f''(r)}{2}-\frac{f(r)u''(r)}{2u(r)}-\frac{f'(r)u'(r)}{u(r)}-\frac{f'(r)}{r}-\frac{f(r)u'(r)}{ru(r)}=\Omega(R), \tag{14}$$

$$-\frac{f''(r)}{2}-\frac{3f(r)u''(r)}{2u(r)}-\frac{f'(r)u'(r)}{u(r)}+\frac{3f(r)u'(r)^2}{2u(r)^2} -\frac{f'(r)}{r}-\frac{f(r)u'(r)}{ru(r)}=\Omega(R), \tag{15}$$

$$-\frac{f(r)u''(r)}{2u(r)}-\frac{f'(r)u'(r)}{2u(r)}-\frac{f'(r)}{r}-\frac{2f(r)u'(r)}{ru(r)}-\frac{f(r)}{r^2}+\frac{1}{r^2}=\Omega(R). \tag{16}$$

By suitably combining these equations, two independent differential equations governing the metric functions can be isolated. The first is a second-order nonlinear differential equation for $u(r)$

$$\frac{u''}{u}-\frac{3}{2}\left(\frac{u'}{u}\right)^2=0, \tag{17}$$

and the second is;

$$-\frac{f''(r)}{2}-\frac{f'(r)u'(r)}{2u(r)}+\frac{f(r)u'(r)}{ru(r)}+\frac{f(r)}{r^2}-\frac{1}{r^2}=0. \tag{18}$$

Solving the first equation yields the general form of the auxiliary function:

$$u(r)=\frac{4}{\left(C_1r+C_2\right)^2}, \tag{19}$$

where C_1 and C_2 are integration constants. In our notation, we denote the arbitrary integration constants that arise throughout the analysis as $C_1, C_2, \ldots, C_n$, with $n \in \mathbb{Z}^+$. Substituting this solution into the second differential equation (18), the metric function $f(r)$ can be obtained as:

$$f(r)=-\frac{\Lambda}{3}r^2+\left(\frac{2C_1}{C_2}+3C_1^2C_3\right)r-\frac{2M}{r}+3C_1C_2C_3+1, \tag{20}$$

where an effective cosmological constant term is defined as

$$\Lambda=-3C_4, \tag{21}$$

that controls the asymptotic (AdS-like) behavior. The inverse radial term $\propto 1/r$ resembles mass effects

$$M=-\frac{C_2^2C_3}{2}, \tag{22}$$

which implies $C_3<0$. The linear term $\propto r$ and constant terms is a direct result of the extended gravitational dynamics and encode genuine deviations from general relativity.

In the limit $C_1=0$, for which the function $u(r)$ (19) reduces to a constant, all non-standard contributions vanish and the metric function (20) smoothly reduces to the Sch–(A)dS solution, confirming the consistency of the model,

$$f(r)=-\frac{\Lambda}{3}r^2-\frac{2M}{r}+1, \tag{23}$$

where $\Lambda>0$ corresponds to the de Sitter background, while $\Lambda<0$ yields the anti–de Sitter geometry. Furthermore, under this constraint, choosing $C_2=2$ leads to the identification of the auxiliary metric with the space-time metric, $u_{\mu\nu}=g_{\mu\nu}$, so that the theory reduces to a single-metric description in which all geometric and physical properties are governed solely by $g_{\mu\nu}$, as in standard general relativity.

The horizon radius is determined by the condition $f\left(r_h\right)=0$ which yields a cubic equation for r. Solving this equation reveals one real root and two complex conjugate roots. The physically relevant event horizon corresponds to the unique real solution (is also called outer event horizon), which is expressed in terms of the cosmological constant Λ and the integration constants as:

$$r_h=\frac{-C_2^2\mathcal{A}^2+2C_2\left(3C_1^2C_2C_3+2C_1\right)\mathcal{A}}{2\Lambda C_2^2\mathcal{A}}\\ \frac{-4\left(3\Lambda C_1C_2^3C_3+9C_1^4C_2^2C_3^2+\Lambda C_2^2+12C_1^3C_2C_3+4C_1^2\right)}{2\Lambda C_2^2\mathcal{A}}, \tag{24}$$

where the auxiliary function $\mathcal{A}$ is used to write r_h in compact form;

$$\mathcal{A}=\left\{-\frac{4}{C_2^3}\left(54C_1^6C_2^3C_3^3+27\Lambda C_1^3C_2^4C_3^2+108C_1^5C_2^2C_3^2+3\Lambda^2C_2^5C_3\right.\right.\\ +27\Lambda C_1^2C_2^3C_3+72C_1^4C_2C_3+6\Lambda C_1C_2^2+\Lambda C_2^2\\ \left.\left.\times\sqrt{\begin{array}{l}(3C_1^3C_2C_3+\Lambda C_2^2+3C_1^2)(27C_1^3C_2^3C_3^3\\ +9\Lambda C_2^4C_3^2+27C_1^2C_2^2C_3^2-4)\end{array}}+16C_1^3\right)\right\}^{1/3}. \tag{25}$$

The existence of real roots for $f(r)=0$ is primarily governed by the term under the square root within the auxiliary function $\mathcal{A}$ which imposes a well-defined constraint on the integration constants and the cosmological constant Λ. Furthermore, by evaluating the metric function at the event horizon, the mass parameter M can be uniquely expressed in terms of the horizon radius and the integration constants as:

$$M=-\frac{\Lambda}{6}r_h^3+\left(\frac{3C_1^2C_2C_3+2C_1}{2C_2}\right)r_h^2+\left(\frac{1+3C_1C_2C_3}{2}\right)r_h. \tag{26}$$

To illustrate the physical viability of the solution, we consider two representative numerical configurations,

$$\left\{C_2=1, C_3=-2\right\} \quad \text{and} \quad \left\{C_2=-1, C_3=-2\right\}, \tag{27}$$

chosen such that the mass parameter is normalized to $M=1$ (22) for convenience. Throughout the analysis, we fix the cosmological constant to $\Lambda=-0.03$, corresponding to an AdS background, and use the Sch–AdS case as a reference. The admissible range of the integration constant C_1 is determined by requiring the reality of the square root term appearing in the auxiliary function (25). For the configurations above, this requirement yields the following intervals for which the horizon equation admits a unique real and positive root:

$$\{-\infty,-0.1853\},\{-0.0919,0.1138\},\{0.4781,\infty\}, \tag{28}$$

and

$$\{-\infty,-0.4781\},\{-0.1138,0.0919\},\{0.1853,\infty\}. \tag{29}$$

respectively. The numerical evaluation of the central admissible intervals confirms that the corresponding horizon radius satisfies $r_h>0$ consistently resides within the range $1.69 \lesssim r_h \lesssim 2.41$ (in close agreement with the Sch-AdS horizon radius $r_h^{Sch} \approx 1.93$). In contrast, the remaining mathematical intervals do not yield physically meaningful configurations when compared to the Sch–AdS case. To examine the influence of the C_1 corrections on the horizon topology, we focus our analysis on the intermediate regime, specifically

$$C_1 \in \{-0.08,-0.03,0.03,0.08\}. \tag{30}$$

Within this range, the curvature singularity at $r=0$ is consistently cloaked by an event horizon, ensuring a well-defined black hole geometry.

Fig. 1 illustrates the behavior of the metric function $f(r)$ as a function of the horizon radius r_h for two configurations in (27). For the considered parameter space, $f(r)$ admits a unique root at $r=r_h$, identifying a single event horizon. At large scales, $f(r)$ recovers the Sch–AdS asymptotic structure, where the integration constant C_1 modulates the horizon radius without introducing additional roots or geometric pathologies.

Finally, combining the conformal ansatz in (6) with the definition (3), one obtains that $p(R)=1+\frac{\epsilon}{4}R$, then using the definition of $u(r)$ (12), this relation allows one to straightforwardly obtain

$$F_R=-\frac{1}{\alpha}\left(\frac{\epsilon}{4}R-u(r)+1\right), \tag{31}$$

then integrating this expression with respect to R, the corresponding curvature function is derived as

$$F(R)=-\frac{\epsilon}{8\alpha}R^2+\frac{1}{\alpha}(u(r)-1)R+C_5. \tag{32}$$

To elucidate the physical content of the theory, we consider the expansion of the action (1) in the limit where the coupling parameter ϵ is small ($\epsilon \to 0$). First, we expand the determinant inside the square root for the first part in Eq. (1), and substitute this back into the first part of the action:

$$\frac{1}{\epsilon}\left[\sqrt{-|g_{\mu\nu}+\epsilon R_{\mu\nu}|}-\lambda\sqrt{-g}\right] \approx \sqrt{-g}\left[\frac{1-\lambda}{\epsilon}+\frac{1}{2}R+\frac{\epsilon}{8}R^2-\frac{\epsilon}{4}R_{\mu\nu}R^{\mu\nu}\right]. \tag{33}$$

Next, we taking into account of the exact solution of $F(R)$, the action (1) finally takes the following form,

$$S \approx \int d^4x\sqrt{-g}\left[\frac{u(r)}{2}R+\frac{\epsilon}{16}R^2-\frac{\epsilon}{4}R_{\mu\nu}R^{\mu\nu}+\Lambda\right]+S_m. \tag{34}$$

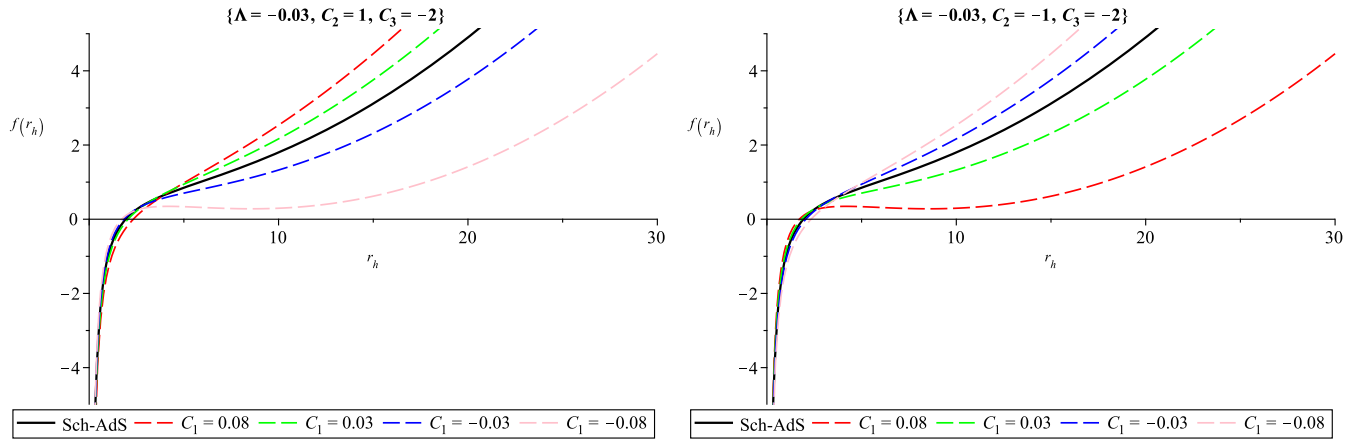

Fig. 1. (color online) The metric function $f(r)$ (20) is plotted as a function of the horizon radius r_H (24) for the set of two configurations in (27). The Sch–AdS (black line) solution (23) shown for reference.

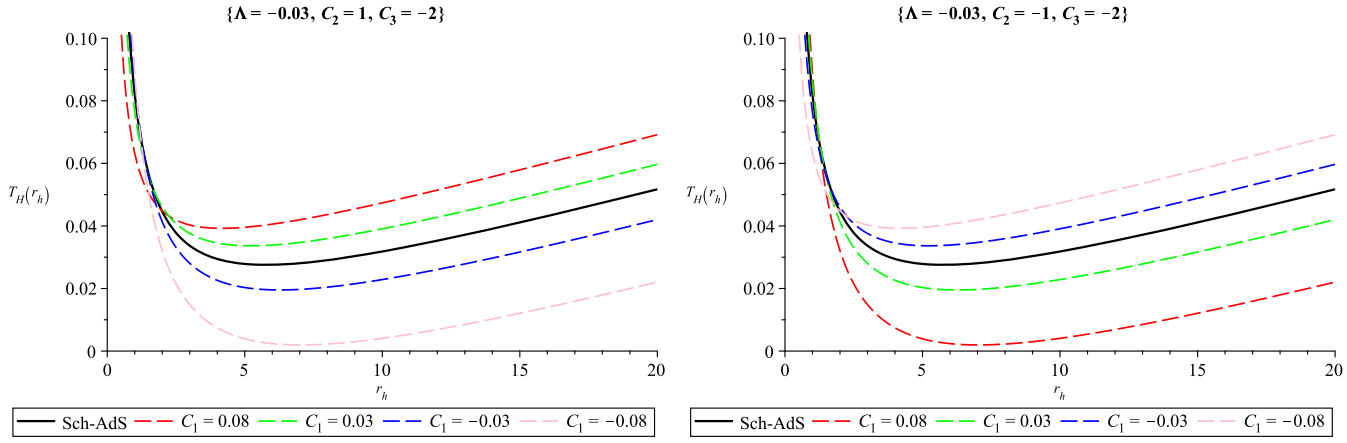

Fig. 2. (color online) The Hawking temperature (20) is plotted as a function of the horizon radius r_H (24) for the parameter sets in (27). The Sch–AdS (black line) solution (23) shown for a reference.

Consequently, the theory reduces to Einstein gravity with an effective cosmological constant, supplemented by quadratic curvature corrections characteristic of BI–type modifications. The function $u(r)$ (19) reflects a nontrivial effective gravitational coupling induced by the modified theory, and the effective cosmological constant emerges (for the previous definition, see Eq. (21)) as a combination of the model parameters (ϵ, α and λ) and integration constant C_5,

$$\Lambda = \frac{1-\lambda}{\epsilon} + \frac{\alpha C_5}{2}. \tag{35}$$

ensuring consistency with the exact solution.

3.1. Singularity analysis from curvature invariants

To determine the regularity of the space-time described by the metric function in (20), we examine curvature scalars constructed from the Riemann tensor. The presence of a physical curvature singularity is characterized by the divergence of the Ricci scalar R, the Ricci tensor squared $R_{\mu\nu}R^{\mu\nu}$, and the Kretschmann scalar $\mathcal{K} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. For the present solution, these invariants take the form;

$$R = 4\Lambda - \frac{18C_1^2C_3 - 12C_1C_2^{-1}}{r} - \frac{6C_2C_1C_3}{r^2}, \tag{36}$$

$$R_{\mu\nu}R^{\mu\nu} = 4\Lambda^2 - \frac{8\Lambda\left(4C_1^2C_3 + 3C_1C_2^{-1}\right)}{r} + \frac{120C_1^3C_2^{-1}C_3 + 90C_1^4C_3^2 + 40C_1^2C_2^{-2} - 12C_2C_1C_3\Lambda}{r^2} + \frac{72C_2C_1^3C_3^2 + 48C_1^2C_3}{r^3} + \frac{18C_2^2C_1^2C_3^2}{r^4}, \tag{37}$$

and

$$\mathcal{K} = \frac{8\Lambda^2}{3} - \frac{8\Lambda\left(3C_1^3C_3 + 2C_1C_2^{-1}\right)}{r} + \frac{96C_1^3C_2^{-1}C_3 + 72C_1^4C_3^2 + 32C_1^2C_2^{-2} - 8C_1C_2C_3\Lambda}{r^2} + \frac{72C_2C_1^3C_3^2 + 48C_1^2C_3}{r^3} + \frac{36C_2^2C_1^2C_3^2}{r^4} + \frac{24C_2^3C_1C_3^2}{r^5} + \frac{12C_2^4C_3^2}{r^6}. \tag{38}$$

Now, let us analyze the asymptotic behavior of these quantities in the limit $r \to 0$. The Ricci scalar diverges as $R \sim r^{-2}$, while the Ricci tensor squared contains terms diverging up to r^{-4}. Most notably, the Kretschmann scalar exhibits the strongest divergence,

$$\mathcal{K} \sim \frac{12C_2^4C_3^2}{r^6}, \quad r \to 0 \tag{39}$$

which is of the same leading order as the Sch-AdS cases. This shows that the central singularity is of Schwarzschild type, characterized by a strong r^{-6} divergence. Subleading divergences reflect the modified gravitational structure but do not alter the leading singular behavior. Conversely, at large radii $r \to \infty$, all terms depending on the integration

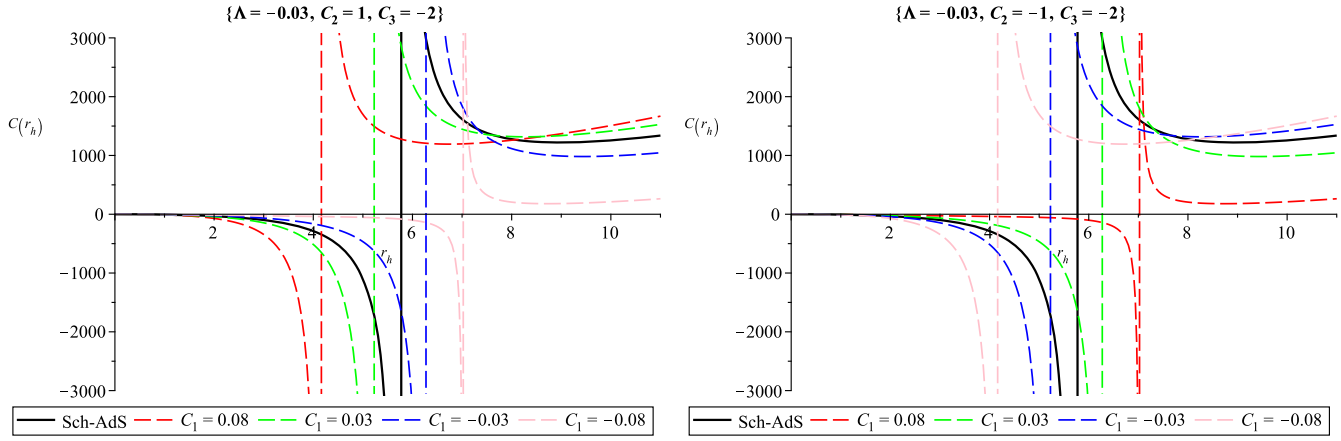

Fig. 3. (color online) The heat capacity C (42) as a function of the horizon radius r_H under the considered parameter sets in (27). The plots include comparisons with the Sch-AdS (black solid line) black hole. The vertical divergences in C signal phase transitions, while the sign of C indicates thermodynamic stability.

constants C_i decay, and the invariants approach; $R \to 4\Lambda$, $R_{\mu\nu}R^{\mu\nu} \to 4\Lambda^2$, and $\mathcal{K} \to \frac{8}{3}\Lambda^2$ confirming that the space-time is asymptotically AdS.

4. Thermodynamics of the model

In this section, we investigate the thermodynamic properties of the black hole solution derived from the BI-$f(R)$ gravitational framework. Thermodynamic analysis provides valuable insights into the interplay between gravity, quantum field theory, and the fundamental laws of black hole mechanics. We begin our thermodynamical analysis by computing the surface gravity, κ, which quantifies the gravitational acceleration at the event horizon and serves as a key quantity in the determination of black hole temperature. For a static and spherically symmetric space-time, the surface gravity is defined as [37]:

$$\kappa = \lim_{r\to r_H}\left(\frac{1}{2}\frac{1}{\sqrt{g_{tt}g_{rr}}}\frac{\partial}{\partial r}g_{tt}\right). \tag{40}$$

In the context of the metric given in (10), and taking into account of the mass expression (26), this expression simplifies to:

$$\kappa = \lim_{r\to r_H}\left(\frac{1}{2}\frac{\partial}{\partial r}g_{tt}\right) = -\frac{\Lambda r_H}{2} + \frac{\left(3C_1C_2C_3+1\right)}{2r_H} + C_1\left(\frac{2}{C_2}+3C_1C_3\right). \tag{41}$$

For the parameter values considered in (27) and (30), the surface gravity evaluated at the event horizon is strictly positive, ensuring that the Hawking temperature T_H is well defined. The Hawking temperature, which characterizes the thermal radiation emitted by the black hole as predicted by quantum field theory in curved space-time [38], is proportional to the surface gravity κ evaluated at the outer horizon $r = r_h$ and is given by

$$T_H = \frac{\kappa}{2\pi} = \frac{\hbar}{4\pi k_B}\left[-\Lambda r_H + \frac{\left(3C_1C_2C_3+1\right)}{r_H} + 2C_1\left(\frac{2}{C_2}+3C_1C_3\right)\right]. \tag{42}$$

Fig. 2 illustrates the thermodynamic behavior of black holes in our modified gravity theory, showing the Hawking temperature (T_H) (42) versus the horizon radius (r_H) for the parameter sets considered in this work (27). All solutions display the characteristic Hawking-Page transition [39], where T_H reaches a minimum, separating unstable (small r_h) from stable (large r_h) black holes.

The entropy associated with the black hole horizon can be determined from the first law of black hole thermodynamics. For a static configuration, the first law reduces to

$$dM = T_H dS, \tag{43}$$

where S denotes the entropy associated with the horizon. Considering the mass M (26) and the Hawking temperature T_H (42) as functions of the horizon radius r_h, one finds

$$dS = \frac{1}{T_H}\frac{dM}{dr_h}dr_h. \tag{44}$$

Integrating with respect to r_h, the entropy is obtained as

$$S(r_h) = \pi r_h^2. \tag{45}$$

In Palatini $F(R)$ theories of gravity, the black hole entropy is generally expected to receive a correction proportional to the derivative $F_R = \frac{dF(R)}{dR}$, leading to the generalized relation $S = \frac{A}{4}F_R$ [40,41], and, in the presence of BI–type extensions, to an even more complex Wald entropy functional [42]. In contrast, the entropy obtained from the thermodynamic integration in the present BI–$F(R)$ framework obeys the standard Bekenstein–Hawking form [43,44] $S = A/4$ (where the horizon area is $A = 4\pi r_h^2$), with no explicit contribution arising from neither the BI sector nor the $F(R)$ modification. This result can be understood as a consequence of the specific structure of the BI–$F(R)$ theory in the Palatini formulation, where the scalar curvature is determined algebraically by the field equations and does not represent an independent dynamical degree of freedom at the horizon.

To assess local thermodynamic stability, we compute the specific heat C at constant horizon radius:

$$C = T\left(\frac{\partial S}{\partial T}\right)_{T=T_H}. \tag{46}$$

Substituting the expressions for S and T_H, we obtain:

$$C = -2\pi r_h^2\left(\frac{2C_1\left(3C_1^2C_2C_3+2\right)r_h + C_2\left(-\Lambda r_h^2 + 3C_1C_2C_3 + 1\right)}{C_2\left(\Lambda r_h^2 + 3C_1C_2C_3 + 1\right)}\right). \tag{47}$$

The sign of the specific heat C is a diagnostic tool for thermodynamic stability: a positive value indicates local stability, while a negative value suggests instability and possible phase transitions.

In Fig. 3, we present the variation of the specific heat capacity as a function of r_H, highlighting the conditions under which the black hole becomes thermodynamically stable or unstable. The analysis reveals that the stability behavior is strongly dependent on the integration constants. For instance, variations in the integration constant C_1 shift the location of these divergences and alter the size of the stable branches, showing that the model parameters significantly influence the stability structure of the black holes.

Moreover, divergences in the heat capacity C are associated with second-order phase transitions, which are often linked to critical points in black hole thermodynamics. Such divergences arise when the denominator of Eq. (47) vanishes, leading to the analytic condition,

$$\Lambda r_c^2 + 3C_1C_2C_3 + 1 = 0, \tag{48}$$

which defines the critical horizon radius,

$$r_c = \sqrt{-\frac{3C_1C_2C_3+1}{\Lambda}}. \tag{49}$$

In the AdS regime ($\Lambda < 0$), the critical radius is real provided $3C_1C_2C_3 + 1 > 0$, a condition satisfied by the parameter sets considered here. Evaluating the Hawking temperature at $r_h = r_c$ then yields the corresponding critical temperature, which characterizes the onset of the thermodynamic phase transition,

$$T_H(r_c) = \frac{\hbar}{2\pi k_B}\left[3C_1^2C_3 + \frac{2C_1}{C_2} + \frac{(3C_1C_2C_3+1)}{\sqrt{-\frac{3C_1C_2C_3+1}{\Lambda}}}\right]. \tag{50}$$

Requiring the critical temperature to be non-negative, $T_H(r_c) \geq 0$, imposes physical bounds on the integration constant C_1. Solving (50) with respect to C_1, for the two configurations considered in (27), the allowed numerical ranges for C_1 are found to be $C_1 > -0.0848$ and $C_1 < 0.0848$, respectively, in accordance with Eqs. (28) and (29). These bounds define the thermodynamically consistent parameter space for which the solution describes a physically viable black hole with a well-defined thermal state. For instance, adopting the first configuration with $C_1 = 0.08$, we obtain $r_c \approx 4.16$, in excellent agreement with the minima of the Hawking temperature shown in Fig. 2, thereby confirming that the divergence of the heat capacity coincides with the transition from negative to positive specific heat.

5. Conclusion

In this work, we have investigated static, spherically symmetric black hole solutions in the framework of BI–$f(R)$ gravity, a class of modified theories motivated by BI–type nonlinearities and extensions of general relativity. Our analysis focused on constructing an exact black hole geometry and examining its geometric and thermodynamic properties within an asymptotically AdS background.

The resulting metric function (20) preserves the asymptotically AdS structure of the space-time, as illustrated in Fig. 1. The integration constant C_1 parametrizes deviations from the Sch–AdS geometry, inducing quantitative shifts in the horizon structure while smoothly recovering the Sch–AdS limit as $C_1 \to 0$. Although BI–type gravity theories are often associated with singularity regularization, the black hole solutions obtained in this work do not eliminate the central curvature singularity. In particular, the Kretschmann scalar diverges as r^{-6} (39), indicating a Sch-type curvature singularity and geodesic incompleteness. This result highlights that the regularizing properties of BI–like modifications are not universal and depend sensitively on the underlying gravitational dynamics.

The Hawking temperature (42) remains positive and exhibits a minimum as a function of the horizon radius, separating small and large black hole branches in close analogy with the Sch–AdS case (see Fig. 2). This behavior signals the onset of thermodynamic stability for sufficiently large black holes. The heat capacity (47) further supports this interpretation: its sign change distinguishes unstable and stable phases, while its divergence marks second-order phase transitions (see Fig. 3). Variations of C_1 shift the critical points but preserve the overall phase structure, confirming that the modified gravity effects introduce controlled deviations without altering the qualitative thermodynamic behavior of the black hole solutions.

In summary, BI-$f(R)$ gravity admits asymptotically AdS black hole solutions with well-defined thermodynamic behavior closely resembling that of Sch–AdS black holes, while exhibiting distinctive quantitative modifications governed by the integration constants. These results provide a consistent starting point for further studies, including extensions to charged or rotating configurations, higher-dimensional generalizations, and potential implications for modified gravity phenomenology.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author used ChatGPT and Gemini in order to improve readability of the text. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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