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An area matching process to estimate the hydraulic parameters using transient constant-head test data

A. Ufuk Şahin and Emin Çiftçi

ABSTRACT

A new parameter estimation methodology was established for the interpretation of the transient constant-head test to identify the hydrogeological parameters of an aquifer. The proposed method, referred as the area matching process (AMP), is based on linking the field data to the theoretical type curve through a unique area computed above these curves bounded by a user specified integration interval. The proposed method removes the need of superimposition of theoretical type curves and field data collected during the test, which may lead to the unexpected errors in assessing aquifer parameters. The AMP approach was implemented for a number of synthetically generated hypothetical test data augmented with several random noise levels, which mimic the uncertainty in site measurement together with porous media heterogeneity, and to an actual field data set available in the literature. The estimation performance of the AMP method was also compared with the existing traditional and recently developed techniques. As demonstrated by the conducted test results, the accuracy, reliability, robustness and simplicity of the proposed technique provide significant flexibility in field applications.

Key words | area matching process, constant-head injection, parameter estimation, transmissivity

A. Ufuk Şahin (corresponding author)
Department of Civil Engineering,
Hacettepe University,
Ankara 06800,
Turkey
E-mail: *ufuksahin@hacettepe.edu.tr*

Emin Çiftçi
Department of Civil Engineering,
Maltepe University,
Istanbul 34857,
Turkey

INTRODUCTION

A constant-head test is a hydrogeological tool, in which groundwater is extracted by maintaining a constant head in a well, and the aquifer parameters (i.e. hydraulic conductivity, storativity) are estimated from the transient discharge measurements. Compared to the constant-head test, the constant-discharge test is a more frequently utilized technique for investigating the hydrogeological properties (Kruseman & de Ridder 1990). However, the use of traditional constant-discharge test may not be convenient, especially in a low-transmissivity formation, where it leads to the occurrence of a deep cone of drawdown in the vicinity of the pumping well which results in the dewatering of the pumping well in a very short period, therefore, adequate measurements cannot be gathered from the nearby observation wells (Freeze & Cherry 1979). On the other hand, this phenomenon is not encountered in the constant-head test, since a fixed water level is maintained at the well; hence this test is more suitable for aquifers of low-transmissivity. Wellbore storage effect is also needed to be considered as an additional unknown parameter in constant-discharge test, whereas this parameter is of no concern in constant-head tests (Mucha & Paulikova 1986). Since the head at the well is maintained at a constant level during the test, the head change in the well would not lead to any water release from wellbore storage (Bundschuh & Suárez 2010).

The first mathematical model describing the flow behavior under a constant-head test was provided by Jacob & Lohman (1952). The temporal variation of discharge is formulated by a cumbersome well function in this pioneering solution which involves a number of assumptions: (i) infinite-extent aquifer with horizontal flow; (ii) fully penetrating well; (iii) homogeneous and isotropic aquifer; and (iv) two-dimensional flow. Some analytical treatments for the problem at hand were proposed by Hantush (1959),

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Glover (1978) and Perrochet (2005). In a more recent work, Chang & Yeh (2009) developed an analytical solution for the constant-head test performed at a partially penetrating well.

Hydraulic parameters from the constant-head test data are traditionally estimated from a curve matching process by superimposing the theoretical type curve ($G(\alpha)$ vs α) and the field data ($Q(t)$ vs t) on a log-log scale (Jacob & Lohman 1952). Once the curves are overlapped, a matching point, selected anywhere on the superimposed curves, yields the corresponding $Q(t)$, $G(\alpha)$, α and t. Using these estimated values, transmissivity (T) and storativity (S) are easily computed. However, an appropriate visual fit may not be attained in practice, as a result of measurement errors during data collection, the paucity in the collected test data, or the presence of heterogeneity. Alternatively, Lohman (1972) suggested a straight-line approximation which is similar to the method of Cooper & Jacob (1946) developed for a constant-discharge aquifer test. This approximation is valid for late-time data, however, there is no criterion describing the critical time when the late-time period begins, which may result in the misinterpretation of the test (Batu 1998).

Apart from these traditional techniques, there are a number of studies in the literature that focused on the simplification of the well function given by Jacob & Lohman (1952) with which the formation properties could be investigated. Glover (1978) proposed a fit equation to simplify the well function given by Jacob & Lohman (1952). Swamee *et al.* (2000) introduced a least-square minimization technique to estimate the aquifer parameters using the function proposed by Glover (1978). Ojha (2004) approximated the Jacob & Lohman (1952) well function by utilizing algebraic manipulations and presented an error minimization process to obtain aquifer parameters. Singh (2007) developed an alternative approximate well function form and formulated a Marquardt algorithm-based optimization method for the estimation of aquifer parameters. Singh (2009) also suggested an optimization scheme coupled with a Kernel method to discretize the time span into a number of uniform time steps for the purpose of assessing formation parameters.

Several techniques are readily available to be used for the estimation of aquifer parameters to analyze the classical constant-discharge data which would be collected from different porous medium formations. Derivative-based techniques developed for constant-discharge pump tests (i.e. Chow 1952; Sen 1986; Copty *et al.* 2011), for instance, could also be adapted to constant-head test to estimate the hydraulic properties. The well function given by Jacob & Lohman (1952) has, however, a sophisticated manner for performing derivative-based aquifer-parameter assessing methods to simplify the estimation procedure. Renard *et al.* (2009) and Avci *et al.* (2013) discussed the shortcomings of time derivation of the available field data for parameter estimation, since the time-derivative of the field data would result in highly erratic results due to some probable abnormalities in collected data. Recently, integration-based aquifer-parameter estimation methodologies were proposed by Avci *et al.* (2011) and Sahin (2012) to smooth the unreliable behavior appearing in derivative-based methods.

The purpose of this study is to formulate a simple parameter estimation method, as simple as the conventional curve matching procedure, to be used in lieu of other optimization based techniques, mentioned above. A new integration based parameter estimation methodology, called area matching process (AMP), was introduced to estimate aquifer parameters. This new technique (AMP) was established on an idea that an area within an integration window will be identical for both the theoretical type curve and the temporal flow rate curve obtained from the site measurements. The application procedure of AMP is straightforward, does not require any complicated calculations or curve matching process to acquire aquifer parameters. The AMP seems to have promising features to estimate aquifer parameters in a reliable, accurate and robust manner when compared to existing techniques as discussed in the following sections.

METHODOLOGY OF AREA MATCHING PROCESS APPROACH

Jacob & Lohman (1952) introduced a mathematical model for transient constant-head test for a fully penetrating, finite-diameter well in a non-leaky homogeneous isotropic formation, as schematically illustrated in Figure 1. The proposed model, which aims to estimate the transmissivity and the storativity of the formation, is derived from the following

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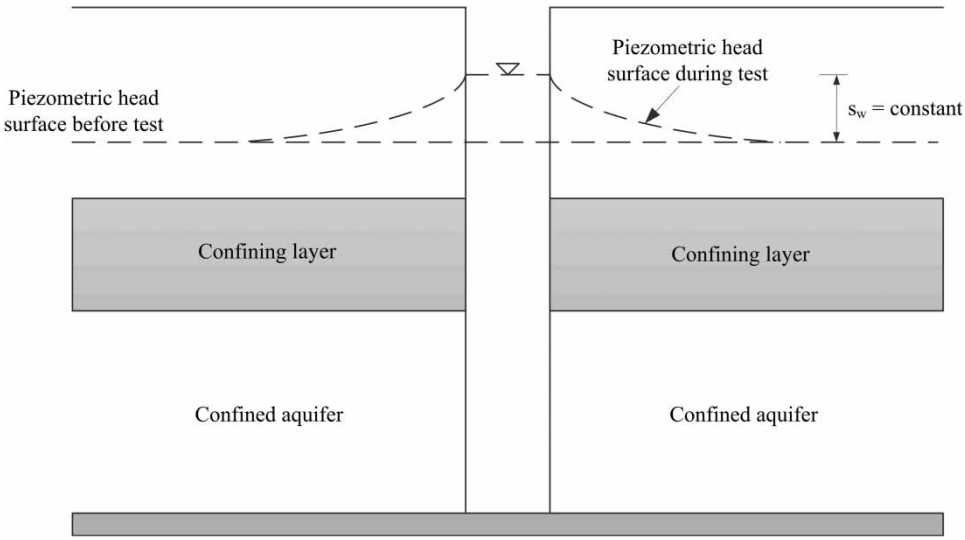


Figure 1 | Schematic representation of constant-pressure head response (redrawn from Batu (1998)).

governing equation:

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}=\frac{S}{T}\frac{\partial s}{\partial t} \tag{1}$$

with the initial and boundary conditions given as

$$\begin{aligned} &s(r,0)=0, && r_w \le r \le \infty \\ &s(r_w,t)=0, && t<0 \\ &s(r_w,t)=s_w=\text{constant} && t\ge 0 \\ &s(\infty,t)=0 && t\ge 0 \end{aligned} \tag{2}$$

where T stands for the aquifer transmissivity, S is the storativity, t is time, r represents the radial coordinate, $s$is the change in head, s_w is the constant head maintained at the well, r_w denotes the radius of the well.

The solution for the time-dependent discharge in a constant-head injection test proposed by Jacob & Lohman (1952) is as follows:

$$Q(t)=2\pi T s_w G(\alpha) \tag{3}$$

where

$$G(\alpha)=\frac{4\alpha}{\pi}\int_0^\infty x\exp(-\alpha x^2)\left\{\frac{\pi}{2}+\tan^{-1}\left[\frac{Y_0(x)}{J_0(x)}\right]\right\}dx \tag{4}$$

and

$$\alpha=\frac{Tt}{Sr_w^2} \tag{5}$$

in which $J_0(x)$ and $Y_0(x)$ are the Bessel functions of zero order of the first and second kinds, respectively.

Jacob & Lohman (1952) tabulated the $G(\alpha)$ for numerous values of α. Based on these values, Perrochet (2005) proposed a simplified equation which gives the dependence of $G(\alpha)$ on α as

$$G(\alpha)\cong\frac{1}{\ln(1+\sqrt{\pi\alpha})} \tag{6}$$

The proposed AMP approach was devised to overcome the aforementioned difficulties in the conventional techniques as explained in the Introduction. This technique shares the same logic behind the curve matching method, but it simplifies the entire process by linking a specified area within an integration window (Δ) for both actual field curve (Q vs. t) and the theoretical curve ($G(\alpha)$ vs. α) as illustrated in Figure 2(a) and 2(b), respectively. The best possible match between these curves is assumed to be realized when the corresponding areas obtained within the same integration window (the shaded regions in Figure 2) are identical.

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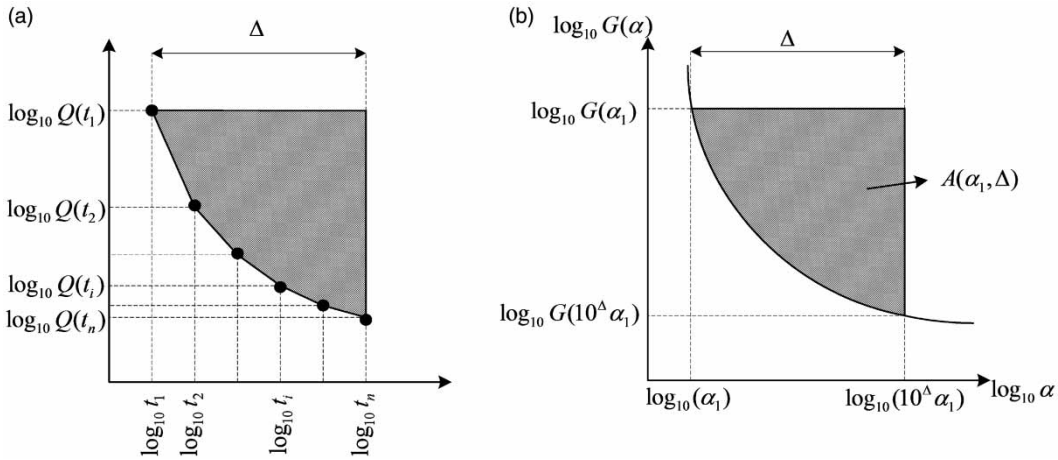


Figure 2 | Graphical illustration of AMP approach; (a) field data, (b) theoretical type curve.

The area of interest for the field curve, shown in Figure 2(a), can be simply calculated from:

$$A = \frac{1}{2}\sum_{i=1}^{n-1}\left\{\left[\log_{10}\left(\frac{Q(t_1)}{Q(t_i)}\right) + \log_{10}\left(\frac{Q(t_1)}{Q(t_{i+1})}\right)\right]\log_{10}\left(\frac{t_{i+1}}{t_i}\right)\right\} \tag{7}$$

where n is the number of data points to be used. The key parameter for the matching procedure is the integration window, Δ, which gives the logarithmic distance between the initial and the final time levels of the utilized data set:

$$\Delta = \log_{10}\left(\frac{t_n}{t_1}\right) \tag{8}$$

The AMP method can also be implemented with just selecting two observation points ($n = 2$) separated with integration window, Δ, instead of employing the whole data set. Under this condition, Equation (7) reduces to the following simplified form

$$A = \frac{1}{2}\log_{10}\left(\frac{Q(t_1)}{Q(t_2)}\right)\Delta \tag{9}$$

where

$$\Delta = \log_{10}\left(\frac{t_2}{t_1}\right) \tag{10}$$

The corresponding area for the theoretical type curve, as also seen in Figure 2(b), varies as a function of an unknown α_1 value and the integration window, Δ, as follows:

$$A(\alpha_1, \Delta) = \{\log_{10}G(\alpha_1)\}\Delta - \int_{\log_{10}(\alpha_1)}^{\log_{10}(10^{\Delta}\alpha_1)}\{\log_{10}G(\alpha)\}d(\log_{10}\alpha) \tag{11}$$

The basic objective of AMP is to estimate an appropriate value for α_1 by linking the theoretical area obtained from Equation (11) and the area obtained from field curve calculated by employing either Equation (7) or Equation (9). The definite integral in Equation (11) can be approximated by a number of alternative numerical integration schemes. In this particular study, the Composite Trapezoidal Rule is employed to evaluate the integral in concern. Figure 3 demonstrates how α_1 varies with A, for some representative Δ values.

The dependence of α_1 on A can be generalized by generating an exponential fit model that has the following form:

$$\log_{10}(\alpha_1) = a\exp(b\log_{10}(A)) + c\exp(d\log_{10}(A)) \tag{12}$$

where a, b, c and d are the coefficients of the fit equation varying with the value of the integration window, Δ. The fit parameters together with the fit statistics are tabulated in Table 1.

Once α_1 value is obtained, the aquifer transmissivity can be acquired by substituting $G(\alpha_1)$ and $Q(t_1)$ into

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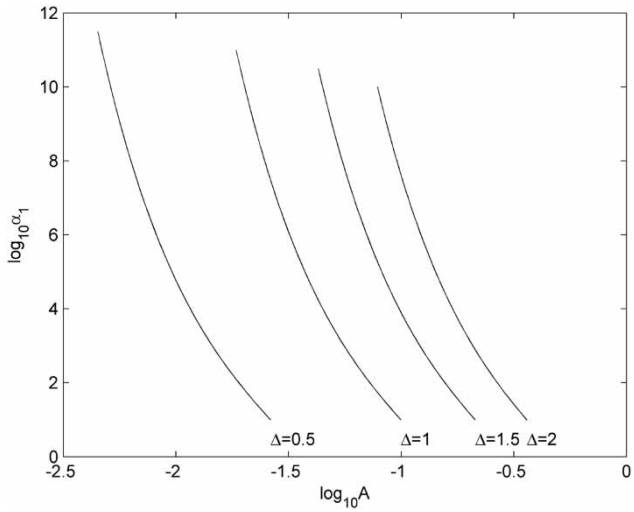


Figure 3 | The variation of α_1 with A.

Equation (3) as

$$\hat{T} = \frac{Q(t_1)}{2\pi s_w G(\alpha_1)} \tag{13}$$

Knowing the predicted transmissivity $\hat{T}$, and substituting Equation (13) into Equation (5), the storativity of the aquifer can be estimated as

$$\hat{S} = \frac{\hat{T} t_1}{\alpha_1 r_w^2} \tag{14}$$

The AMP approach could be summarized with the following key steps:

Step 1. The test data set is substituted in Equation (7) to obtain the area, A, that is to be matched. The integration window, Δ, is evaluated by Equation (8). If only two observation points are employed, Equations (9) and (10) can be employed to get A and Δ, respectively.

Step 2. Based on the value of the integration window, Δ, the fit parameters are determined from Table 1 and these parameters with the calculated A value are then substituted in Equation (12) to compute α_1.

Step 3. α_1 is substituted in Equation (4) (or in Equation (6)) to get the corresponding $G(\alpha_1)$.

Step 4. Finally, $G(\alpha_1)$ and $Q(t_1)$ are substituted into Equation (13) to get the aquifer transmissivity, T, which in turn, is substituted into Equation (14) to obtain the aquifer storativity, S.

RESULTS AND DISCUSSION

The performance of AMP was investigated through the numerical experiments conducted with different aquifer settings. The reliability and the performance of the proposed method were first examined with synthetically generated test data; to which, random noise was introduced in order to mimic the effects of possible measurement errors, fluctuations in the discharge rate, as well as the heterogeneity of the formation on the available test data. The contribution of the integration window on the estimation performance of AMP was also shown. In addition, the proposed method was implemented in numerical experiments, in which six different heterogeneous aquifer scenarios were simulated, in order to elaborate the variation of estimated transmissivity values. Finally, AMP was assessed with a real field test data set from the hydrogeology literature.

Test case 1: Synthetic data with random noise

A number of numerical experiments were conducted to simulate different homogeneous aquifer settings, characterized by a wide range of transmissivity values varying from 10^{-6} to 10^{6} m^2/min (log-uniformly spaced 61 transmissivity values were employed). For all the scenarios, storativity, S, was selected as 10^{-4}. The test data were collected from a discharge well with a finite-radius of 0.1 m. The well was assumed to be subjected to a constant drawdown of 10 m throughout the test in order to observe transient discharge response of the hypothetical aquifer. The tests were designed to last from 1 to 100 min with a uniform logarithmic time step of 0.1 min. In addition, two different levels of random noise, being in the order of 1%, and 2% of the observed discharge, were added to each generated data set to imitate the possible error sources (i.e. measurement errors, discharge fluctuations, or heterogeneity) encountered during the test. Thus, the AMP procedure was executed for a total of 122 distinct data sets (61 different Ts $\times 2$ different random noise orders). Figure 4 portrays one of the generated synthetic data sets with different random noise levels.

In these hypothetical test cases, for simplicity, the analyses were undertaken via selecting two points from

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Table 1 | The coefficient of fit form and statistical properties in Equation (12)

	Parameters				Statistics			
Δ	*a*	*b*	*c*	*d*	SSE	R^2	R^2 Adj.	RMSE
0.1	−1955.34	2.628738	0.001667	−2.36566	0.000438	1	1	0.002032
0.2	−350.522	2.559461	0.006826	−2.36966	0.000444	1	1	0.002057
0.3	−128.262	2.49452	0.015523	−2.37359	0.000448	1	1	0.002075
0.4	−63.3889	2.433597	0.027759	−2.37747	0.000449	1	1	0.002087
0.5	−37.0788	2.37642	0.043526	−2.38131	0.000448	1	1	0.002095
0.6	−24.175	2.322705	0.062815	−2.3851	0.000445	1	1	0.002098
0.7	−17.0047	2.272198	0.085613	−2.38887	0.00044	1	1	0.002098
0.8	−12.6507	2.224659	0.111912	−2.39262	0.000434	1	1	0.002094
0.9	−9.82577	2.179913	0.1417	−2.39635	0.000427	1	1	0.002088
1	−7.89589	2.137756	0.174969	−2.40007	0.000419	1	0.999999	0.002079
1.1	−6.52199	2.098013	0.21171	−2.40378	0.000411	1	0.999999	0.002068
1.2	−5.51039	2.060518	0.251916	−2.4075	0.000401	1	0.999999	0.002055
1.3	−4.74444	2.025144	0.29558	−2.41122	0.000391	1	0.999999	0.002041
1.4	−4.15061	1.991764	0.342694	−2.41495	0.000381	0.999999	0.999999	0.002025
1.5	−3.68076	1.960239	0.393254	−2.4187	0.000371	0.999999	0.999999	0.002008
1.6	−3.30244	1.930456	0.447254	−2.42247	0.00036	0.999999	0.999999	0.001989
1.7	−2.99315	1.902338	0.504688	−2.42627	0.000349	0.999999	0.999999	0.00197
1.8	−2.73685	1.875786	0.565552	−2.43009	0.000338	0.999999	0.999999	0.00195
1.9	−2.52189	1.850709	0.629843	−2.43395	0.000327	0.999999	0.999999	0.001929
2	−2.33966	1.827029	0.697556	−2.43785	0.000316	0.999999	0.999999	0.001907
2.1	−2.18368	1.804697	0.768686	−2.44178	0.000305	0.999999	0.999999	0.001884
2.2	−2.04898	1.783623	0.843234	−2.44576	0.000294	0.999999	0.999999	0.001861
2.3	−1.93172	1.763771	0.921195	−2.44979	0.000283	1	0.999999	0.001837
2.4	−1.82889	1.745063	1.002569	−2.45387	0.000273	1	0.999999	0.001813
2.5	−1.7381	1.727479	1.087352	−2.45801	0.000262	1	0.999999	0.001787

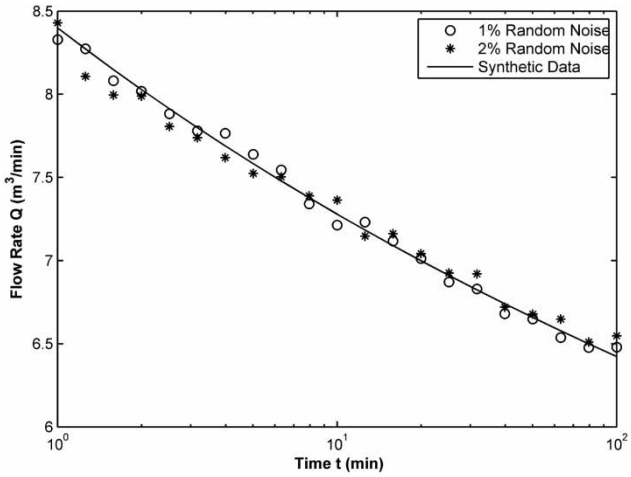


Figure 4 | Flow rate response of synthetic data (generated $T = 1\ m^2/min$, $S = 1 \times 10^{-4}$).

each data set. As explained in the previous section, the use of more data points inside the integration window would yield more accurate area approximations. The AMP procedure was followed for all available distinct data sets to obtain the estimation results as depicted in Figure 5 and tabulated in Table 2. The integration intervals employed in this numerical experiment represent different phases of the test data of interest as given in Table 2.

The estimated transmissivity values obtained from AMP were compared with the generation values in order to analyze the performance of the proposed method. Root mean squared error (RMSE) and the coefficient of determination (R^2) were employed in comparison. RMSE between the

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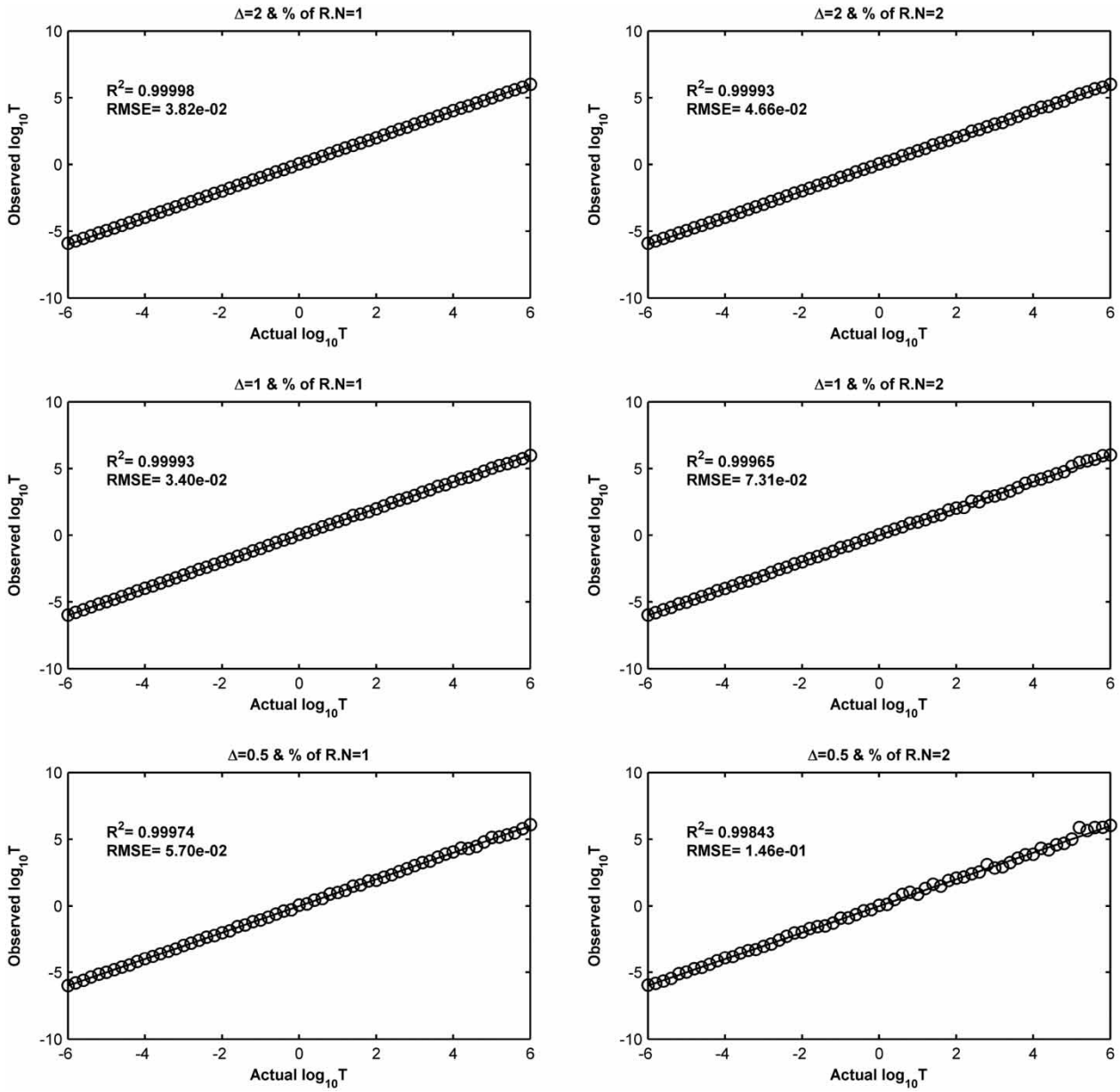


Figure 5 | The estimation performance of AMP.

observed and actual data pair can be calculated as

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2} \tag{15}$$

where N is the total number of data, $\hat{y}_i$ is the estimated data value and y_i is the actual data value. R^2 value can be given as

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{N}\left(y_i - \frac{1}{N}\sum_{i=1}^{N} y_i\right)^2} \tag{16}$$

As seen through the results, summarized in Table 2, the estimated transmissivity values match quite well

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Table 2 | AMP performance for noisy data set

	Random noise 1%				Random noise 2%			
Integration window Δ	RMSE	R^2	Max. absolute error	Min. absolute error	RMSE	R^2	Max. absolute error	Min. absolute error
2	0.038236	0.99998	0.10198	0.025218	0.046597	0.99993	0.1045	0.04391
1.9	0.036664	0.99998	0.095022	0.01829	0.047807	0.99991	0.11212	0.051904
1.8	0.035968	0.99997	0.086503	0.020317	0.043687	0.99994	0.10625	0.053386
1.7	0.032834	0.99997	0.071714	0.035084	0.038576	0.99994	0.087383	0.039419
1.6	0.031393	0.99997	0.067473	0.028837	0.043244	0.9999	0.092132	0.064931
1.5	0.035054	0.99996	0.07251	0.04861	0.046852	0.99988	0.11187	0.08252
1.4	0.032485	0.99996	0.080356	0.052118	0.049636	0.99987	0.13125	0.066414
1.3	0.030346	0.99997	0.074189	0.036482	0.060511	0.99974	0.14698	0.092399
1.2	0.025799	0.99997	0.061173	0.03712	0.057813	0.99977	0.16323	0.091952
1.1	0.035	0.99993	0.078924	0.069239	0.063327	0.99975	0.19781	0.089668
1	0.033992	0.99993	0.089567	0.072161	0.073062	0.99965	0.25116	0.10891
0.9	0.035418	0.99992	0.079144	0.07767	0.080256	0.99956	0.3049	0.11918
0.8	0.041939	0.99987	0.12012	0.072232	0.10196	0.99936	0.32052	0.15949
0.7	0.04468	0.99985	0.17545	0.073699	0.094512	0.99931	0.31285	0.19458
0.6	0.049217	0.99981	0.15348	0.11415	0.10597	0.9991	0.3755	0.17737
0.5	0.056996	0.99974	0.14103	0.13023	0.1462	0.99843	0.67589	0.25183

with the predefined ones, in almost all scenarios as shown in Figure 5. The effect of integration window on the estimation performance of the proposed AMP was also investigated in terms of error metrics such as R^2 and RMSE. For both noise levels, the RMSE values increase when the integration window is narrowed, as demonstrated in Figure 6. For instance, the RMSE statistics for the estimated transmissivity values are 3.82×10^{-2}, 3.40×10^{-2}, and 5.69×10^{-2}, when the data sets with 1% noise level are analyzed within integration windows of 2, 1 and 0.5, respectively. The same error trend could also be observed for the data sets with 2% of random noise level. The primary reason for such an error trend lies behind the fact that the approximated area is more sensitive to fluctuations when the integration window is narrow, whereas larger integration windows smooth the influence of fluctuations. Figure 6 also implies that using integration windows with different sizes would not significantly improve the estimation performance of AMP in terms of RMSE values for a smooth data set (i.e. for random noise level 1%). R^2 values in Table 2 show that any arbitrarily chosen integral interval could provide the accurate estimation results which are in good accordance with the generated values. This error metric implies that integration interval does not

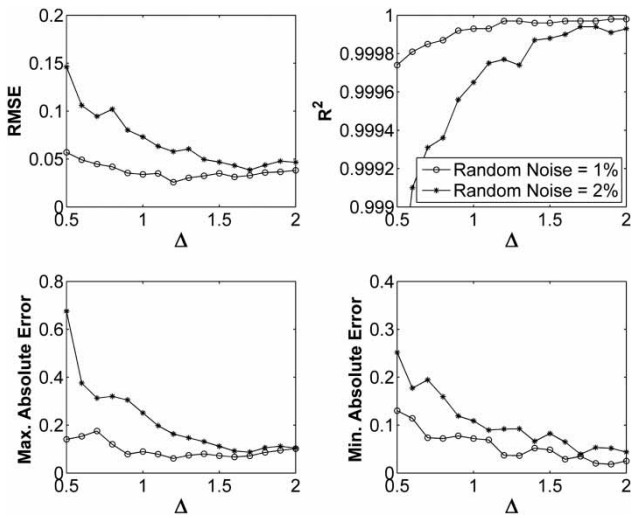


Figure 6 | The variation of error metrics with integration window.

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significantly improve the estimation results. In other words, each integration window could be preferred to identify hydrogeologic properties.

Eventually, regardless of which integration window is preferred, these hypothetical experiments verify that for homogeneous systems, the proposed AMP is capable of estimating aquifer parameters with a high accuracy even if the collected data suffer from substantial noise.

Test case 2: Synthetic data analyses for heterogeneous field

A set of numerical experiments, in which spatially variable transmissivity fields were employed, were conducted with PMWIN-MODFLOW (Chiang & Kinzelbach 2001) to test the performance of the proposed AMP in heterogeneous medium. To achieve this goal, six separate log-normally distributed transmissivity fields were generated by the Turning Band Algorithm, TBM (Mantoglou & Wilson 1982) using two different integral scale (I = 5 m, 20 m) and three different variance ($\sigma^2 = 0.5, 1, 2$) values for the purpose of simulating different heterogeneity conditions. All transmissivity fields were assumed to have a geometric mean of 1 m^2/day ($T_g = 1\ m^2$/day). A pumping well was placed at the center of a 499 m × 499 m square domain which was discretized with square cells of size $1\,m \times 1\,m$. The aquifer storativity was assumed to be uniform being equal to 10^{-5}. The simulations were performed for a period of 100 minutes in order to eliminate potential boundary effects. A uniform time discretization scheme was employed with 1,000 time steps. Constant-head condition was assigned to the pumping well cell; therefore, a constant drawdown of 10 m was maintained at the well throughout the simulations. The calculated flow rate values for each time step were read from the output water-budget file generated by PMWIN-MODFLOW. Temporal variations of synthetically generated discharge data for the given six alternative scenarios are pictured in Figure 7.

Using the generated discharge data, the proposed model was applied to successive data pairs corresponding to the time levels t_1 and $t_2 = 10^{\Delta} t_1$, where the integration window, Δ, was preferred to be 1. The estimated transmissivity was seen to vary with the selected t_1 value as illustrated in Figure 8.

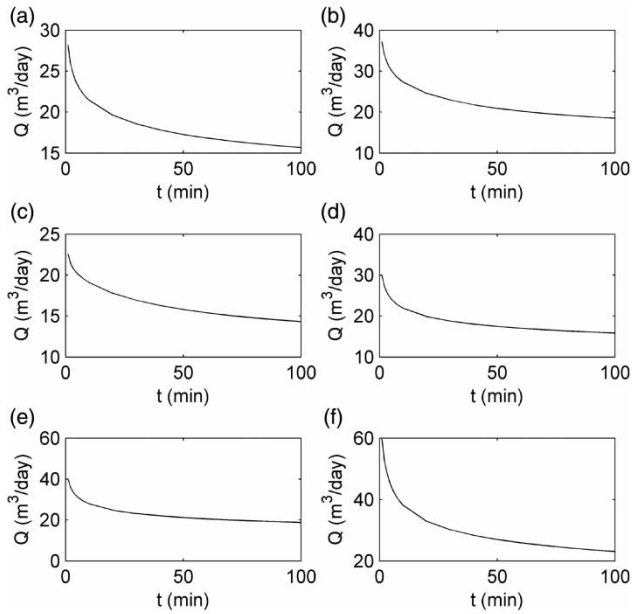


Figure 7 | Temporal variation of discharge for the test case with: (a) I = 5 m & $\sigma^2 = 0.5$; (b) I = 5 m & $\sigma^2 = 1$; (c) I = 5 m & $\sigma^2 = 2$; (d) I = 20 m & $\sigma^2 = 0.5$; (e) I = 20 m & $\sigma^2 = 1$; and (f) I = 20 m & $\sigma^2 = 2$.

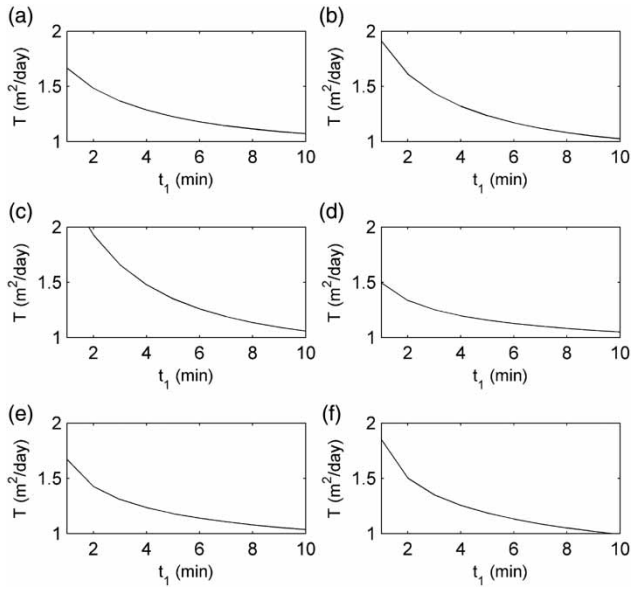


Figure 8 | Variation of estimated transmissivity with t_1 for the test case with: (a) I = 5 m & $\sigma^2 = 0.5$; (b) I = 5 m & $\sigma^2 = 1$; (c) I = 5 m & $\sigma^2 = 2$; (d) I = 20 m & $\sigma^2 = 0.5$; (e) I = 20 m & $\sigma^2 = 1$; and (f) I = 20 m & $\sigma^2 = 2$.

In all cases, the estimated transmissivity value was observed to approach to the geometric mean of the transmissivity field as t_1 increases, as shown in Figure 8.

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The previous studies on the conventional constant-discharge pump tests in the confined heterogeneous aquifer show that the interpreted transmissivity estimates eventually yield to some weighted average of the transmissivity values inside the cone of depression which expands with elapsed time from the beginning of pumping (i.e. Bibby 1979; Meier *et al.* 1998; Sanchez-Vila *et al.* 1999; Copty & Findikakis 2004; Wu *et al.* 2005; Sanchez-Vila *et al.* 2006; Copty *et al.* 2011; Avci & Sahin 2014). The same trend, the interpreted transmissivity values at the late time of constant discharge tests leading to the geometric mean of a normally distributed transmissivity field, was therefore verified by this test case example based on the available results.

Table 3 | Lohman (1965) data

Observation # *i*	Time *t* (min)	Discharge *Q* (m^3/min)
1	1	0.02756
2	2	0.02626
3	3	0.02604
4	4	0.02377
5	5	0.02354
6	6	0.02354
7	8	0.02252
8	11	0.02214
9	16	0.02142
10	21	0.02082
11	26	0.02021
12	31	0.02021
13	41.5	0.01976
14	51	0.01951
15	61	0.01934
16	76	0.01911
17	91	0.01892
18	103	0.01862

Test case 3: Lohman data

The performance of the proposed AMP approach was also verified with a real field data set example of a constant-drawdown test obtained from Artesia Heights well near Grand Junction, Colorado in 1948 (Lohman 1965). The radius of the well of interest (r_w) and the constant-head maintained at the well (s_w) were reported to be 0.084 m and 28.14 m, respectively. The test results are tabulated in Table 3.

As a first experiment, the whole data set, that lies within an integration window, Δ, being approximately equal to 2, was utilized. The proposed methodology described in previous sections produced the output given in Table 4. To verify the accuracy of the method, the estimated transmissivity and storativity values were substituted in Equation (3) to regenerate the discharge values. The calculated and the observed discharge values are seen to be in good agreement, as can be seen in Figure 9.

As a second phase, the estimation performance of the proposed method was compared with its alternatives, which are the conventional Curve-matching Method and the Straight-line Method (Lohman 1972), Glover's (1978) method, and the relatively new techniques developed by Swamee *et al.* (2000) and Singh (2007). The aquifer parameters provided by these alternative techniques are presented in Table 5. The regenerated discharge values with these estimated formation parameters, as well as the actual discharge data are exhibited in Figure 10. In addition to the picture portrayed in Figure 10, the RMSE statistics depicted in Table 5 noticeably demonstrate the advantage of the proposed method in terms of its accuracy.

As a final remark, the capability of AMP was tested with only two observation points utilized in the estimation process. The implementation of the AMP as described in the methodology section obviously becomes much more simplified once the number of employed data is reduced to two. Figure 11 demonstrates the results of four different

Table 4 | The application of AMP with results

		Fit parameters							
A	Δ	a	b	c	d	α	G(α)	T (m^2/min)	S
0.178502	2	−2.33966	1.827029	0.697556	−2.43785	5345.233	0.208434	0.000748	1.98×10^{-5}

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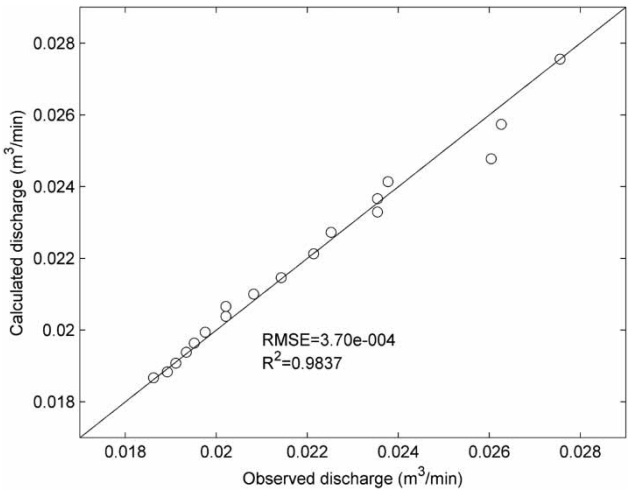


Figure 9 | Comparison of observed and calculated flowrates.

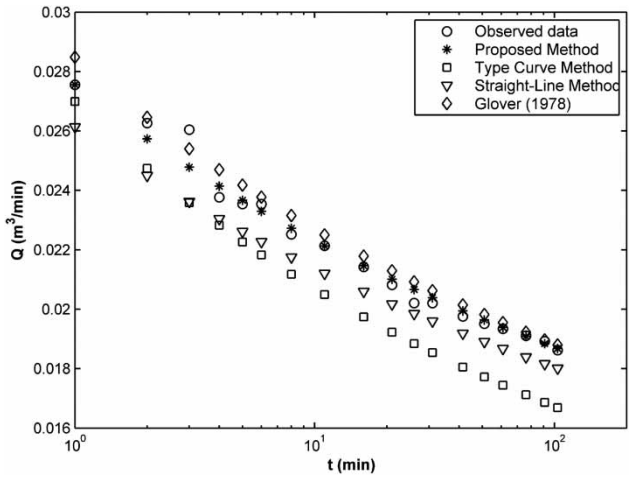


Figure 10 | Calculated discharge variations for the implemented methods.

Table 5 | Estimation performance of implemented methods

	T	*S*	RMSE
Proposed AMP	7.48×10^{-4}	1.98×10^{-5}	3.70×10^{-4}
Type-Curve Method (Batu 1998)	5.58×10^{-4}	1.58×10^{-4}	1.67×10^{-3}
Straight-Line Method (Batu 1998)	7.49×10^{-4}	1.15×10^{-5}	1.05×10^{-3}
Glover (1978)	7.11×10^{-4}	4.15×10^{-5}	5.02×10^{-4}
Swamee *et al.* (2000)	6.96×10^{-4}	3.88×10^{-5}	3.92×10^{-4}
Singh (2007)	7.02×10^{-4}	3.64×10^{-5}	3.71×10^{-4}

experiments conducted with four randomly selected pairs of data points. Even with two data points, the AMP was seen to produce quite satisfactory results, which is another advantage of the proposed method.

Figure 11 also shows the AMP would lead to almost the same transmissivity estimates (as a mean of 7.53×10^{-4}m^2/min), which seems independent from whichever data phase is employed to obtain aquifer parameters. In this regard, the results of AMP for the Lohman (1965) data are in line with those of synthetically generated test data studied in Test case 1. The AMP, therefore, could provide practitioners a great flexibility in the data analysis. A definite late-time criterion was not described by the Lohman (1972) straight-line method as discussed in Batu (1998). This potential drawback of the straight-line method can be overcome by performing AMP as delineated in Figure 11.

CONCLUSIONS

In this work, a new parameter estimation procedure, named as AMP, was formulated to analyze the transient constant-head data. The proposed AMP is as straightforward as the classical curve matching techniques; however, it avoids the need for superimposing the theoretical and the field curves. The idea behind this approach is founded on matching a unique area within an integration interval, which is indeed a common property for both the theoretical type curve and the field discharge curve. The reliability and the performance of the AMP were tested for various synthetically generated data sets augmented with random noise, as well as a real field example.

The following key conclusions can be drawn from these test cases. (i) The size of the integration window does not significantly influence the estimation performance of AMP. This feature implies that the available site data can be directly used to estimate aquifer parameters, without any need for interpolation or extrapolation that may lead to the misinterpretation of test results. (ii) For homogeneous aquifer settings, no matter which data phase (early- or late-time response) is utilized in the parameter estimation process; the AMP is able to estimate the hydrologic properties accurately, which in turn, provides a great flexibility to field practitioners while assessing aquifer parameters. (iii) For heterogeneous aquifer systems, the use of late time data was noticed to provide transmissivity values close to

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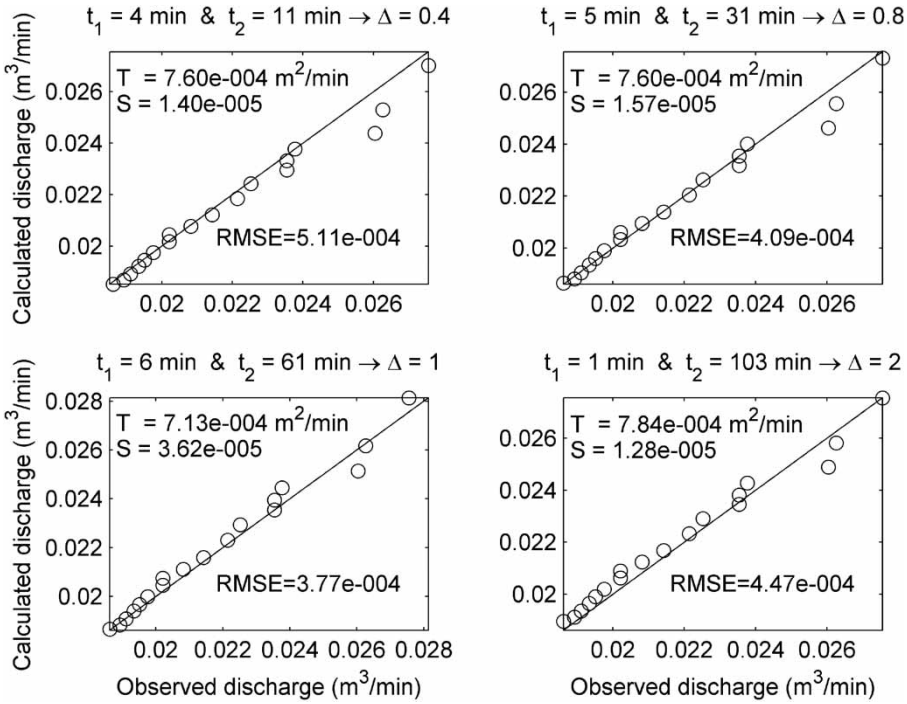


Figure 11 | Estimation performance of AMP with $n = 2$.

the geometric mean of a normally distributed transmissivity field. (iv) AMP is still applicable even with the use of two observation points, which allows this method to be utilized in circumstances when there is lack of sufficient data. (v) And finally, the parameter estimation capability of the AMP is as high as those of the present alternative methods.

In the light of these findings, the AMP can be evaluated as a viable approach for the interpretation of the transient constant-head test results. In order to simplify and automatize the curve-matching process, the established methodology based on area matching has the potential to be extended to other aquifer types such as leaky or unconfined aquifers.

REFERENCES

Avci, C. B. & Sahin, A. U. 2014 Assessing radial transmissivity variation in heterogeneous aquifers using analytical techniques. *Hydrological Processes* **28**, 5739–5754.

Avci, C. B., Sahin, A. U. & Ciftci, E. 2011 Aquifer parameter estimation using an incremental area method. *Hydrological Processes* **25**, 2584–2596.

Avci, C. B., Sahin, A. U. & Ciftci, E. 2013 A new method for aquifer system identification and parameter estimation. *Hydrological Processes* **27**, 2485–2497.

Batu, V. 1998 *Aquifer Hydraulics: A Comprehensive Guide to Hydrogeologic Data Analysis*. Wiley, New York.

Bibby, R. 1979 *Estimating Sustainable Yield to a Well in Heterogeneous Strata*. Alberta Research Council Bulletins 37, Edmonton, p. 60.

Bundschuh, J. & Suárez, M. C. A. 2010 *Introduction to the Numerical Modeling of Groundwater and Geothermal Systems: Fundamentals of Mass, Energy and Solute Transport in Poroelastic Rocks*. CRC Press, Taylor & Francis, London.

Chang, Y. C. & Yeh, H. D. 2009 New solutions to the constant-head test performed at a partially penetrating well. *Journal of Hydrology* **369**, 90–97.

Chiang, W. H. & Kinzelbach, W. 2001 *3D-Groundwater Modeling with PMWIN a Simulation System for Modeling Groundwater Flow and Pollution*. Springer, Berlin, New York, Heidelberg.

Chow, V. T. 1952 On the determination of transmissibility and storage coefficients from pumping test data. *Transactions – American Geophysical Union* **33**, 397–404.

Cooper, H. H. J. & Jacob, C. E. 1946 A generalized graphical method for evaluating formation constants and summarizing well-field history. *Transactions of the American Geophysical Union* **27**, 526–534.

Copty, N. K. & Findikakis, A. N. 2004 Stochastic analysis of pumping test drawdown data in heterogeneous geologic formations. *Journal of Hydraulic Research* **42**, 59–67.

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Copty, N. K., Trinchero, P. & Sanchez-Vila, X. 2011 Inferring spatial distribution of radially integrated transmissivity from pumping tests in heterogeneous confined aquifers. *Water Resources Research* **47**, W05526, DOI:10.1029/2010WR009877.

Freeze, R. A. & Cherry, J. A. 1979 *Groundwater*. Prentice-Hall, Englewood Cliffs, NJ.

Glover, R. E. 1978 *Transient Ground Water Hydraulics*. Water Resources Publications, Fort Collins, CO.

Hantush, M. S. 1959 Nonsteady flow to flowing wells in leaky aquifers. *Journal of Geophysical Research* **64**, 1043–1052.

Jacob, C. E. & Lohman, S. W. 1952 Nonsteady flow to a well of constant drawdown in an extensive aquifer. *Transactions – American Geophysical Union* **334**, 559–569.

Kruseman, G. P. & de Ridder, N. A. 1990 *Analysis and Evaluation of Pumping Test Data*, 2nd edn. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands.

Lohman, S. W. 1965 *Geology and Artesian Water Supply of the Grand Junction Area Colorado*. Professional Paper No. 451, US Geological Survey, Washington, DC.

Lohman, S. W. 1972 *Ground-water Hydraulics*. Professional Paper No. 708, US Geological Survey, Washington, DC, pp. 23–27.

Mantoglou, A. & Wilson, J. L. 1982 The Turning Bands Method for simulation of random fields using line generation by a spectral method. *Water Resources Research* **18**, 1379–1394.

Meier, P. M., Carrera, J. & Sanchez-Vila, X. 1998 An evaluation of Jacob's method for the interpretation of pumping tests in heterogeneous formations. *Water Resources Research* **34**, 1011–1025.

Mucha, I. & Paulikova, E. 1986 Pumping test using large diameter production and observation wells. *Journal of Hydrology* **89**, 157–164.

Ojha, C. S. P. 2004 Aquifer parameters estimation using artesian well test data. *Journal of Hydrologic Engineering* **91**, 64–67.

Perrochet, P. 2005 A simple solution to tunnel or well discharge under constant drawdown. *Hydrogeology Journal* **13**, 886–888.

Renard, P., Glenz, D. & Mejias, M. 2009 Understanding diagnostic plots for well-test interpretation. *Hydrogeology Journal* **17**, 589–600.

Sahin, A. U. 2012 *Incremental Area Method Approach for the Determination of Aquifer Parameters and Aquifer Systems Identification*, PhD Thesis, Department of Civil Engineering, Institute for Graduate Studies for Science and Engineering, Boğaziçi University, Turkey.

Sanchez-Vila, X., Meier, P. M. & Carrera, J. 1999 Pumping tests in heterogeneous aquifers: an analytical study of what can be obtained from their interpretation using Jacob's method. *Water Resources Research* **35**, 943–952.

Sanchez-Vila, X., Guadagnini, A. & Carrera, J. 2006 Representative hydraulic conductivities in saturated groundwater flow. *Reviews of Geophysics* **44**, RG3002.

Sen, Z. 1986 Determination of aquifer parameters by the slope matching method. *Ground Water* **24**, 217–223.

Singh, S. K. 2007 Simple approximation of well function for constant drawdown variable discharge artesian wells. *Journal of Irrigation and Drainage Engineering* **133**, 282–285.

Singh, S. 2009 Kernel method for transient rate and volume of well discharge under constant drawdown. *Journal of Irrigation and Drainage Engineering* **135**, 252–256.

Swamee, P. K., Mishra, G. C. & Chahar, B. R. 2000 Simple approximation for flowing well problem. *Journal of Irrigation and Drainage Engineering* **126**, 65–67.

Wu, C. M., Yeh, T. C. J., Lee, T. H., Hsu, N. S., Chen, C. H. & Sancho, A. F. 2005 Traditional aquifer tests: comparing apples to oranges? *Water Resources Research* **41**, W09402.

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